MATH 319, Fall 2013, Assignment 1

Due date: Friday, September 13

Name (printed): ______________________________

UW Student ID Number: ______________________________

Discussion Section: (circle)

Liu Liu:	**301**	**302**	**303**	**304**
Huanyu Wen:	**305**	**306**	**323**	**324**
Dongfei Pei:	**325**	**326**	**329**	
Kai Hsu:	**327**	**328**		

Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.
2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.
3. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

Correctness
/20

Completeness
/5

Total:	/25
Bonus:	/3

Introduction to Differential Equations

Suggested problems:

Section 1.1: 1-33
Section 1.2: 1-19
(Note: It can be easily checked that the DE $y' = ay + b$ *has the general solution* $y(t) = Ce^{at} - b/a$ *where* $C \in \mathbb{R}$*)*

Problems for submission:

Section 1.1: 4, 14, 22
Section 1.2: 5 (only a and b)
(Justify your answers for full marks!)

1. For the following first-order ODEs, do the following: (i) Sketch the *direction/slope field* in the (x, y)-plane; (ii) Determine the solution (by integration); and (iii) Overlay the solutions found in part (ii) onto the slope field found in part (i). (Note: all derivatives are with respect to x!)

   (a) $y' = x^2 - 1$

   (b) $y' = \dfrac{x}{1-x}, x \neq 1$

   (c) $y' = \tan(x), x \in (-\pi/2, \pi/2)$
   (**Hint!** Use the substitution $u = \cos(x)$ to integrate.)

2. Sketch the *direction/slope field* in the (x, y)-plane for the following first-order ODEs. Overlay a few plausible solutions. (Note: You do not have to determine the analytic form of the solutions $y(x)$!)

   (a) $y' = y^2 + 2y + 1$

   (b) $y' = x^2 - y^2$

3. Verify that the following functions $y(x)$ are solutions of the given second-order ODEs and satisfy the given initial conditions.

   (a) $y'' + 2y' + y = 0; y(0) = 1; y'(0) = 1; y(x) = (1 + 2x)e^{-x}$

   (b) $x^2y'' + 3xy' + y = 0; y(1) = 1; y'(1) = 0; y(x) = (\ln(x) + 1)/x$

4. The process of verifying solutions of differential equations is not limited to ordinary differential equations. The one-dimensional heat equation, which governs the diffusion of heat in a one-dimensional medium over time, is a *partial differential equation* given by

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}.$$

Show that

$$u(t,x) = t^{-1/2} e^{-x^2/4t}$$

is a solution of the heat equation.

BONUS! What does the initial (i.e. $t = 0$) temperature profile look like? What is the long-term behavior of the temperature profile? What is the physical interpretation? (**Hint:** Take the limit as $t \to 0^+$ holding x fixed. Consider the case $x = 0$ separately. Note that the profile is not continuous!)