MATH 319, Fall 2013, Assignment 2

Due date: Friday, September 20

Name (printed): ___________________________________

UW Student ID Number: ___________________________________

Discussion Section: (circle)

Liu Liu:	**301**	**302**	**303**	**304**
Huanyu Wen:	**305**	**306**	**323**	**324**
Dongfei Pei:	**325**	**326**	**329**	
Kai Hsu:	**327**	**328**		

Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.
2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.
3. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

Correctness
/20

Completeness
/5

Total:	/25
Bonus:	/3

First-Order Linear and Separable DEs, Existence and Uniqueness

Suggested problems:

Section 2.1: 1-33
Section 2.2: 1-28

Problems for submission:

Section 2.1: 9, 16, 33
Section 2.2: 5, 8, 23
(Justify your answers for full marks!)

1. Consider the first-order ODE $y' = \sqrt{1-y^2}$.

   (a) Sketch the slope/direction field over the range $-\infty < x < \infty$, $-1 \le y \le 1$.

   (b) Find a solution $y(x)$ of the the differential equation over the domain $C - \pi/2 < x < C + \pi/2$ where C is an arbitrary (integration) constant. Notice that $y(x) = -1$ is a solution in the range $x \le C - \pi/2$ and $y(x) = 1$ is a solution in the range $x \ge C + \pi/2$. (**Hint:** The answer you get must match the slope field found in part (a).)

   (c) Apply the existence and uniqueness theorem from class. Comment on any regions where existence and/or uniqueness breaks down. Do the results match those found in parts (a) and (b)? Explain.

2. Consider a body cooling (or warming) under *Newton's law of cooling*, which says that the rate of a body's temperature change (dT/dt) is proportional to the difference between the body's current temperature (T) and the ambient environmental temperature (A). In other words, we have

$$\frac{dT}{dt} = k(A - T), \quad T(0) = T_0 \tag{1}$$

   where $k > 0$ is the proportionality constant.

   (a) Find the solution of the IVP (1) by treating it as a *separable differential equation.*

(b) Find the solution of the IVP (1) by rearranging it as a *first-order linear differential equation.*

(c) Comment on the long-term behavior of the solutions found in parts (a) and (b) (i.e. what happens in the limit $t \to \infty$?). Does the long-term behavior depend on the initial condition?

(d) Suppose the temperature of the external environment is no longer a constant value A but fluctuates sinusoidally with the seasons according to the relationship $A + B\sin\left(\frac{\pi}{6}t\right)$, which gives

$$\frac{dT}{dt} = k\left(A + B\sin\left(\frac{\pi}{6}t\right) - T\right). \tag{2}$$

Here we imagine that t corresponds to the months, so that $t = 0$ corresponds to January, $t = 1$ corresponds to February, and so on. Find the general solution of this DE with the parameter values $A = 50$, $B = 25$, and $k = 1/6$ (temperatures in Fahrenheit).

(e) Comment on the long-term behavior of the solution found in part (d) (i.e. what happens in the limit $t \to \infty$?). Does it depend on the initial condition?

(f) **BONUS!** The solution found in part (d) can be divided into two portions: a *transient* portion which goes to zero in the limit $t \to \infty$; and a *steady state* portion which does not. Identify the transient and steady state portion of the solution $T(t)$. In which month does the steady state portion of the temperature of the body reach its maximum? Minimum? Comment on how these values compare with the maximal and minimal values of the external environment. (**Hint:** Find the critical points of the steady state portion of the solution! They will not be "nice" numbers (Sorry!).)