MATH 319, Fall 2013, Assignment 3

Due date: Friday, September 27

Name (printed): ______________________________

UW Student ID Number: ______________________________

Discussion Section: (circle)

Liu Liu:	**301**	**302**	**303**	**304**
Huanyu Wen:	**305**	**306**	**323**	**324**
Dongfei Pei:	**325**	**326**	**329**	
Kai Hsu:	**327**	**328**		

Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.
2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.
3. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

Correctness
/20

Completeness
/5

Total:	/25
Bonus:	/3

Substitution Methods, Exact DEs

Suggested problems:

Section 2.2: 30-38
Section 2.4: 27-31
Section 2.6: 1-16,18-23,25-32

Problems for submission:

Section 2.2: 32, 35 (parts (a) and (b) only)
Section 2.4: 28
Section 2.6: 12, 28
(Justify your answers for full marks!)

1. Suppose that $n \neq 0$ and $n \neq 1$. Show that the substitution $v = y^{1-n}$ transforms the Bernoulli equation

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

into the first-order linear equation

$$\frac{dv}{dx} + (1-n)P(x)v(x) = (1-n)Q(x).$$

2. Assume that a population (denoted P) grows at a rate proportional to its own population when the population size is small (i.e. proportional to P) but encounters a second-power crowding term when the population is large (i.e. proportional to P^2). This gives rise to the *logistic growth model*

$$\frac{dP}{dt} = kP(M-P), \quad P(0) = P_0 \tag{1}$$

where $k, M > 0$ are non-negative constants.

(a) Sketch a slope field for (1) in the region $t \geq 0, P \geq 0$. Conjecture as to what the long-term behavior of solutions is. (*Hint:* Leave the parameter values undetermined! You should still be able to plot all of the important features.)

(b) Treating the equation as a separable equation, derive the following solution of (1):

$$P(t) = \frac{MP_0}{P_0 + (M - P_0)e^{-kMt}}.$$

(c) Derive the solution by treating the equation as a Bernoulli equation.

(d) By taking the limit as $t \to \infty$, confirm your conjecture about the long-term behavior of solutions.

Bonus! Suppose we add a harvesting term to the logistic equation in the form of a constant value r. This could model, for example, a constant demand for lumber or animal pelts over time. This gives the modified logistic model

$$\frac{dP}{dt} = kP(M - P) - r, \quad P(0) = P_0. \tag{2}$$

(a) Solve the initial value problem (2) with the parameter values $k = 1$, $M = 2$, $r = 1$. (*Hint:* Factor before you integrate!)

(b) Describe the long-term behavior of trajectories in the region $P_0 > 0$ and explain how this differs from that of the model (1). Does this make sense? (*Hint:* Consider the initial conditions $0 < P_0 < 1$, $P_0 = 1$, and $P_0 > 1$ separately. Be careful to note any asymptotes in the solution!)