MATH 319, Fall 2013, Assignment 6

Textbook Questions (2 pages!)

Section 3.5, #10 Find the general solution of the following differential equation:

$$u'' + \omega_0^2 u = \cos \omega_0 t$$

#13 Solve the following initial value problem:

$$y'' + y' - 2y = 2t, \quad y(0) = 0, \quad y'(0) = 1$$

#18 Solve the following initial value problem:

$$y'' + 2y' + 5y = 4e^{-t} \cos 2t, \quad y(0) = 1, \quad y'(0) = 0$$

#23(a) Determine a suitable particular solution $y_p(x)$ trial form if the method of undetermined coefficients is to be used for the following differential equation:

$$y'' - 4y' + 4y = 2t^2 + 4te^{2t} + t \sin 2t.$$

Section 3.6, #4 Use the method of variation of parameters to find a particular solution of the following differential equation. Then check your answer by using the method of undetermined coefficients.

$$4y'' - 4y' + y = 16e^{t/2}$$

#8 Find the general solution of the following differential equation:

$$y'' + 4y = 3 \csc 2t, \quad 0 < t < \pi/2$$

#17 Verify that the following functions y_1 and y_2 satisfy the corresponding homogeneous equation; then find a particular solution of the given nonhomogeneous equation.

$$x^2 y'' - 3xy' + 4y = x^2 \ln x, \quad x > 0; \quad y_1(x) = x^2, \quad y_2(x) = x^2 \ln x.$$

#30 The method of reduction of order (Section 3.4) can also be used for the nonhomogeneous equation

$$y'' + p(t)y' + q(t)y = g(t),$$

provided one solution y_1 of the corresponding homogeneous equation is known. Setting $y = v(t)y_1(t)$, one can show that y satisfies the above equation if v is a solution of

$$y_1(t)v'' + \left[2y_1'(t) + p(t)y_1(t)\right] v' = g(t).$$

This equation is a first order linear equation for v'. Solving this equation, integrating the result, and then multiplying by $y_1(t)$ leads to the general solution of the original equation. Use this method to determine the solution of

$$t^2y'' + 7ty' + 5y = t, \quad t > 0; \quad y_1(t) = t^{-1}$$