MATH 319, Fall 2013, Assignment 7

Due date: Friday, November 1

Name (printed): _____

UW Student ID Number: _____

Discussion Section: (circle)

Liu Liu:	**301**	**302**	**303**	**304**
Huanyu Wen:	**305**	**306**	**323**	**324**
Dongfei Pei:	**325**	**326**	**329**	
Kai Hsu:	**327**	**328**		

Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.
2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.
3. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

Correctness
/20

Completeness
/5

Total:	/25
Bonus:	/3

Suggested problems:

Section 3.7: 1-7, 9-11, 15, 17
Section 3.8: 1-12, 15, 17-25

Problems for submission:

Section 3.7: 1, 4, 11
Section 3.8: 1, 12, 15
(Justify your answers for full marks!)

1. Resonance is not a phenomenon reserved for undamped mechanisms. Consider the following example:

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x(t) = \cos(\omega t). \tag{1}$$

where ω is as yet undetermined. That is to say, suppose we have $m = 1$ kg, $c = 1$ $N/(m/s)$ and $k = 1$ N/m.

   (a) Find the general solution of (1). [**Hint:** Note that we do not need to consider cases for ω!]

   (b) By considering the limit as $t \to \infty$, divide the solution from part (a) into two parts: a *transient solution* $x_{tr}(t)$ which goes to zero in the limit, and a *steady periodic* solution $x_{sp}(t)$ which does not. (In other words, write $x(t) = x_{tr}(t) + x_{sp}(t)$.)

   (c) Find the amplitude of the steady periodic function $x_{sp}(t)$ found in part (c). [**Hint:** Consider writing the portion $x_{sp}(t)$ in the form $A\cos(\omega t - \alpha)$ but only find A.]

   (d) At which value of ω does A achieve its maximum? Interpret this value in terms of the physical system. In particular, how does it compare to the quasi frequency of the unforced system? [**Hint:** Take the derivative of A with respect to ω!]

Bonus! Consider a pendulum/spring modeled by the following equation

$$\frac{d^2x}{dt^2} + c\frac{dx}{dt} + x(t) = \cos(t)$$

where $c \geq 0$ is the unspecified damping coefficient. By using the method of Question #1(a-b), determine the amplitude of the steady periodic portion of the solution as a function of c and find the maximal amplitude. To what type of system does the maximal amplitude correspond?