MATH 319, Fall 2013, Assignment 8

Due date: Friday, November 8

| Name (printed): | |
|-----------------------|--|
| | |
| UW Student ID Number: | |

Discussion Section: (circle)

| Liu Liu: | 301 | 302 | 303 | 304 |
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| Huanyu Wen: | 305 | 306 | 323 | 324 |
| Dongfei Pei: | 325 | 326 | 329 | |
| Kai Hsu: | 327 | 328 | | |

Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.

| Correctness | |
|-------------|-----|
| | /20 |

2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.

| Completeness | |
|--------------|----|
| | /5 |

3. You are encouraged to collaborate on assignment problems but you must write up your assignment independently.

Copying is strictly forbidden!

| Total: | /25 |
|--------|-----|
| Bonus: | /3 |

Suggested problems:

Section 5.1: 1-7, 9-28 Section 5.2: 1-19, 23-28

Problems for submission:

Section 5.1: 2, 7, 23 Section 5.2: 5, 8 (parts (a), (b) and (d) only) (Justify your answers for full marks!)

1. Consider the nonhomogeneous second-order differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 1. (1)$$

- (a) Find the general solution of (1) by using either the method of undetermined coefficients or variation of parameters.
- (b) Find the power series solution of (1). [**Hint:** Due to the non-homogeneity, you will have a different recursion relation on the coefficients for n = 0 and $n \ge 1$. Also note that the recursion does not include the term $a_0!$]
- (c) Rearrange the series found in part (b) to give the general solution found in part (a). [**Hint:** This is challenging! You should, however, be able to identify a piece of the Taylor series expansion of e^x in the series solution in (b). By adding and subtracting terms appropriately to complete the summation, you should be able to obtain the solution in part (a)!]

Bonus! Reconsider the initial value problem

$$\frac{dy}{dx} = x^2 + y^2, \quad y(0) = 0$$

from the first week of class. Find the first four non-trivial terms in the power series and use this to estimate the value of y(1.5). How does this compare with the "true" solution of y(1.5) = 1.517447537? [**Hint:** This differs from the examples in class because we will have to multiply the power series expansions y(x) together to obtain y^2 . This can be accomplished by multiplying the series term-by-term.]