# MATH 319, Fall 2013, Assignment 10

Due date: Wednesday, December 4

Name (printed): ______________________________

UW Student ID Number: ______________________________

Discussion Section: (circle)

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|---|---|---|---|---|
| **Liu Liu:** | **301** | **302** | **303** | **304** |
| **Huanyu Wen:** | **305** | **306** | **323** | **324** |
| **Dongfei Pei:** | **325** | **326** | **329** | |
| **Kai Hsu:** | **327** | **328** | | |

### Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.
2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.
3. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

| Correctness |
|---|
| /20 |

| Completeness |
|---|
| /5 |

| | |
|---|---|
| Total: | /25 |
| Bonus: | /3 |

**Suggested problems:**

Section 7.1: 1-6, 18
Section 7.2: 1,5-7,10,11,20-26
Section 7.3: 16-18, 20, 21

**Problems for submission:**

Section 7.1: 3, 4
Section 7.2: 6(a) and (c), 24
Section 7.3: 17, 20 *(Justify your answers for full marks!)*

1. Consider the following second-order differential equation:

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x(t) = 0. \tag{1}$$

Note that this corresponds to a critically damped pendulum/spring with $m = 1$, $c = 4$, and $k = 4$.

(a) Convert (1) into a system of two first-order differential equations in the variables $x_1(t)$ and $x_2(t)$.

(b) Sketch the vector field diagram of the system found in part (a) in the $(x_1, x_2)$-plane.

(c) Find the general solution of (1) by using the method previously used in class. Use this to construct the solution of the first-order system found in part (a). Verify that this is indeed a solution of the system. (*Hint:* Remember how $x_1(t)$ and $x_2(t)$ are defined!)

**Bonus!** The method by which we have constructed vector field diagrams so far is not restricted to *linear* systems of first-order differential equations. Consider the system of *non-linear* differential equations

$$\frac{dx}{dt} = x^2 - y - 2$$
$$\frac{dy}{dt} = x - y.$$

Sketch the vector field diagram for this sytem. Conjecture as to the long-term behavior of solutions for the initial conditions $(x(0), y(0)) =$

$(0,2)$ and $(x(0), y(0)) = (2,0)$. (*Hint:* Even though the system is more complicated, we will be able to use the method from class to solve determine where $x'(t) = 0$ or $y'(t) = 0$ and subsequently divide the $(x, y)$-plane into regions where all solutions are pushed in the same direction.)

Compare your sketch to a `Maple` plot of the vector field over the range $-2 \leq x \leq 3$, $-2 \leq y \leq 3$. Include a print-out of the plot. The relevant `Maple` code is

```
with(DEtools):
dfieldplot([diff(x(t),t)=x(t)^2-y(t)-2,diff(y(t),t)=x(t)-y(t)],
     [x(t),y(t)],t=0..2,x=-2..3,y=-2..3);
```