MATH 319, Fall 2013, Assignment 10

Textbook Questions

Section 7.5, #3 Find the general solution of the given system of equations and describe the behavior of the solution as $t \to \infty$. Also draw the direction field and plot a few trajectories of the system:

$$\mathbf{x}' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \mathbf{x}$$

#16 Solve the initial value problem and describe the behavior of the solution as $t \to \infty$.

$$\mathbf{x}' = \begin{bmatrix} -2 & 1 \\ -5 & 4 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

#21 The system $t\mathbf{x}' = A\mathbf{x}$ is analogous to the second order Euler equation (Section 5.4). Assuming that $\mathbf{x} = \xi t^r$, where ξ is a constant vector, then ξ and r must satisfy $(A - rI)\xi = \mathbf{0}$ in order to obtain nontrivial solutions of the given differential equation. Use this observation to solve the following system of differential equations.

$$t\mathbf{x}' = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \mathbf{x}$$

Section 7.6, #9 Find the solution of the given initial value problem then describe the behavior of the solution as $t \to \infty$.

$$\mathbf{x}' = \begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Section 7.7, #6 Find a fundamental matrix $\Psi(t)$ of the following given system of equations. Then find the fundamental matrix $\Phi(t)$ satisfying $\Phi(0) = I$.

$$\mathbf{x}' = \begin{bmatrix} -1 & -4 \\ 1 & -1 \end{bmatrix} \mathbf{x}$$

#12 Solve the initial value problem

$$\mathbf{x}' = \begin{bmatrix} -1 & -4 \\ 1 & -1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

by using the fundamental matrix $\Phi(t)$ found in Problem 6.

Section 7.8, #7(a) Find the solution of the given initial value problem.

$$\mathbf{x}' = \begin{bmatrix} 1 & -4 \\ 4 & -7 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Section 7.9, #5 Find the general solution of the given system of equations.

$$\mathbf{x}' = \begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix} \mathbf{x} + \begin{bmatrix} t^{-3} \\ -t^{-2} \end{bmatrix}, \quad t > 0.$$