Math 319, Fall 2013, Term Test II

Techniques in Ordinary Differential Equations

Date: Friday, November 15

Lecture Section: 001

Name (printed): ______________________________

UW Student ID Number: ______________________________

Discussion Section: (circle)

Liu Liu:	**301**	**302**	**303**	**304**
Huanyu Wen:	**305**	**306**	**323**	**324**
Dongfei Pei:	**325**	**326**	**329**	
Kai Hsu:	**327**	**328**		

Instructions

1. Fill out this cover page **completely** and make sure to circle your discussion section.
2. Answer questions in the space provided, using the last page for overflow.
3. Show all the work required to obtain your answers.
4. No calculators are permitted.

FOR EXAMINERS' USE ONLY

Page	Mark
2	/6
3	/7
4	/6
5	/6
Total	/25

1. Short Answer:

[1] (a) Consider the mechanical model $mx''(t) + cx'(t) + kx(t) = 0$. State a relationship on the constants m, c, and k such that the mechanism is *critically damped.*

[2] (b) Determine the Wronskian of $y_1(x) = \ln(x)$ and $y_2(x) = 1/x^2$. Can $y_1(x)$ and $y_2(x)$ constitute a fundamental solution set for a homogeneous second-order differerential equation (for $x > 0$)?

[3] 2. True/False:

(a) The complementary solution of $y''(x) - 8y'(x) + 20y(x) = e^{4x}\sin(2x)$ is $y_c(x) = C_1e^{4x}\sin(2x) + C_2e^{4x}\cos(2x)$. [True / False]

(b) The amplitude (i.e. maximal value) of the particular solution of $y''(x) + 2y'(x) + 5y(x) = \cos(\omega x)$ does not depend on ω. [True / False]

(c) Every point $x_0 \in \mathbb{R}$ is an ordinary point of $(x^2+1)y''(x) + xy'(x) - (x+1)y(x) = 0$. [True / False]

3. Second-Order Differential Equations

[3] (a) Consider the following differential equation:

$$y''(x) + 4y'(x) + 5y(x) = g(x).$$

Determine the complementary solution $y_c(x)$ then set-up, but *do not attempt to evaluate*, the trial form $y_p(x)$ for the given choices of $g(x)$ below.

(i) $g(x) = x^2 e^{-2x}$

(ii) $g(x) = e^{-2x}\cos(x)$

[4] (b) Given that $y_1(x) = 1/x$ and $y_2(x) = 1/x^5$ are fundamental solutions of $x^2y''(x) + 7xy'(x) + 5y(x) = 0$, determine the particular solution of

$$x^2y''(x) + 7xy'(x) + 5y(x) = x.$$

[*Hint:* Remember the standard form for when applying variation of parameters!]

4. Power Series Solutions

[2] (a) Determine the radius of convergence of the power series:

$$\sum_{n=2}^{\infty} \frac{(-1)^n 3^n}{(2n)!}(x-1)^n$$

[2] (b) Write the following expression as a single summation whose common term is x^n:

$$\sum_{n=2}^{\infty}(n-1)n^2 a_n x^{n-2} + x\sum_{n=0}^{\infty} n a_n x^{n-1}$$

[2] (c) Suppose that a particular differential equation has a power series solution centered at $x_0 = 0$ generated by the recursion relation

$$a_{n+2} = \frac{a_{n+1} - (n+1)a_n}{n+2}, \quad n \geq 0.$$

Determine up to the x^3 term of the solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$.

5. Laplace Transforms:

[2] (a) Evaluate the following inverse Laplace transform:

$$\mathcal{L}^{-1}\left\{\frac{2}{s^2-4s+8}\right\}$$

[4] (b) Use the Laplace transform method to evaluate the following initial value problem:

$$\frac{d^2y}{dx^2}-y(x)=e^x, \quad y(0)=0,\ y'(0)=1/2.$$

THIS PAGE IS FOR ROUGH WORK