# MATH 320, Spring 2013, Assignment 2

Due date: Friday, February 8

Name (printed): ______________________________

UW Student ID Number: ______________________________

Discussion Section: (circle)

| | | | | |
|---|---|---|---|---|
| **Robin Prakash:** | **301** | **302** | **303** | |
| **Sowmya Acharya:** | **304** | **306** | **307** | **308** |
| **Raghvendra Chaubey:** | **352** | **353** | **354** | **355** |

## Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.
2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.
3. Show all the work required to obtain your answers.
4. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

| S# | Q# | Mark |
|---|---|---|
| 1.3 | all | /5 |
| 1.4 | all | /10 |
| 1.5 | all | /7 |
| — | 1 | /8 |
| Total: | | /30 |

# Slope Fields, Separable and First-Order Linear Differential Equations

**Suggested problems:**

Section 1.3, 1-22
Section 1.4, 1-29, 32
Section 1.5, 1-25, 31, 32

**Problems for submission:**

Section 1.3: 13, 14, 25
*(You do not have to do the 95% part of question #25)*
Section 1.4: 9, 14, 23, 27
Section 1.5: 2, 19, 24
*(Justify your answers for full marks!)*

1. Consider a body cooling (or warming) under *Newton's law of cooling* (ref: page 2 of text). This says that the rate of a body's temperature change ($dT/dt$) is proportional to the difference between the body's current temperature ($T$) and the ambient environmental temperature ($A$). In other words, we have

$$\frac{dT}{dt} = k(A - T), \quad T(0) = T_0. \tag{1}$$

   (a) Find the solution of the initial value problem (1) by treating it as a *separable differential equation.*

   (b) Find the solution of the initial value problem (1) by rearranging it as a *first-order linear differential equation.*

   (c) Now suppose the temperature of the external environment is no longer a constant value $A$, but fluctuates sinusoidally with time about some mean (i.e. it has the form $A + B\sin(t)$). Using the intuition behind Newton's law of cooling, rewrite (1) using this new form of the ambient environmental temperature.

(d) Solve the initial value problem derived in part (c) with the parameter values $A = 40$, $B = 10$, $k = 1$, and $T_0 = 0$.

(e) Non-homogeneous differential equations frequently arise when there is an external time-dependent force acting on the model, as there is in this case. A common feature of the solutions of such systems is that they can be decomposed into *long-term* and *transient* components (i.e. part of the solution remains for all time, while some only factors sigificantly for a short-term). By considering the limit $t \to \infty$, identify the long-term and transient portions of the solution found in part (d).

(f) Comment on the long-term behavior of the temperature profile of the body (i.e. relate the solution's long-term behavior to the physical problem).