

MATH 320, Spring 2013, Assignment 4

Due date: Friday, March 1

Name (printed): _____

UW Student ID Number: _____

Discussion Section: (circle)

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Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.
2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.
3. Show all the work required to obtain your answers.
4. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

S#	Q#	Mark
1.5	36	/4
2.1	11	/3
2.4	4	/3
2.4	7	/3
2.6	3	/3
2.6	6	/3
—	1	/7
—	2	/4
Total:		/30

Applications, Euler's Method, Runge-Kutta Method

Suggested problems:

Section 1.5: 33-37

Section 2.1: 1-24

Section 2.4: 1-24

Section 2.6: 1-24

Problems for submission:

Section 1.5: 36

Section 2.1: 11

(The relevant model is $dP/dt = \left(\frac{a}{\sqrt{P}} - \frac{b}{\sqrt{P}} \right) P$)

Section 2.4: 4, 7

Section 2.6: 3, 6

(Justify your answers for full marks!)

1. Suppose a factory is built upstream of Lake Mendota (volume 0.5 km^3) which introduces a pollutant into the upstream water system. Suppose the affected water system pumps 0.25 km^3 of water into the lake each year and the downstream water system removes water from the lake at the same rate. Suppose the concentration of pollutant in the feeding water system is 40 kg/km^3 .
 - (a) Set up a first-order differential equation which models the amount of pollutant (in kg) in Lake Mendota.
 - (b) Suppose that there is initially no pollutant in the lake. How much pollutant is in the lake after (i) one month; (ii) seven months; (iii) five years? What is the limiting amount of pollutant in the lake?
 - (c) Suppose now that the inflow and outflow rates of the upstream and downstream water systems vary based on the seasons. Suppose this variance can be modeled by the form $[\text{volume rate in}] = [\text{volume rate out}] = 0.25 + 0.25 \cos(2\pi t)$. Derive the corresponding first-order differential equation which models the amount of pollutant (in kg) in Lake Mendota.
 - (d) Suppose that there is initially no pollutant in the lake. Under the assumptions of part (c), determine how much pollutant is in the lake after (i) one month; (ii) seven months; (iii) five years. What

is the limiting amount of pollutant in the lake? Is it the same as in part (b)? Does it converge to this value faster or slower than part (b)? Provide a brief explanation for the observed differences.

2. Suppose we are modeling an industrial chemical reaction of the form $X \rightarrow Y$ subject to continuous inflow of X and outflow of both X and Y . A simple model for the concentrations $x = [X]$ and $y = [Y]$ is the *system* of first-order differential equations

$$\begin{aligned}\frac{dx}{dt} &= \alpha - \beta x, & x(0) &= x_0 \\ \frac{dy}{dt} &= \gamma x - \delta y, & y(0) &= y_0\end{aligned}$$

where $\alpha, \beta, \gamma, \delta > 0$ and $\gamma > \beta$.

- (a) Solve this differential equation for the parameter values $\alpha = \beta = 1$, $\gamma = \delta = 2$, and $x_0 = y_0 = 0$. [**Hint:** Solve the equation for x first, and then substitute it into the equation for y .]
- (b) What are the (unitless) concentrations of X and Y in the tank at time $t = 0.1$, $t = 1$, and $t = 2$? What are the limiting values of the concentrations?