

MATH 320, Spring 2013, Assignment 4

Textbook Questions

Section 1.5, #36 A tank initially contains 60 gal of pure water. Brine containing 1 lb of salt per gallon enters the tank at 2 gal/min, and the (perfectly mixed) solution leaves the tank at 3 gal/min; thus the tank is empty after exactly 1 h. (a) find the amount of salt in the tank after t minutes. (b) What is the maximum amount of salt ever in the tank?

Section 2.1, #11 Suppose that when a certain lake is stocked with fish, the birth and death rates β and δ are both inversely proportional to \sqrt{P} . (a) Show that

$$P(t) = \left(\frac{1}{2}kt + \sqrt{P_0} \right)^2,$$

where k is a constant. (b) If $P_0 = 100$ and after 6 months there are 169 fish in the lake, how many will there be after 1 year?

Section 2.4 An initial value and its solution $y(x)$ are given. Apply Euler's method twice to approximate this solution on the interval $[0, \frac{1}{2}]$, first with step size $\Delta x = 0.25$, then with step size $\Delta x = 0.1$. Compare the three decimal-place values of the two approximations at $x = \frac{1}{2}$ with the value $y(\frac{1}{2})$ of the actual solution.

#4 $y' = x - y, y(0) = 1; y(x) = 2e^{-x} + x - 1$

#7 $y' = -3x^2y, y(0) = 3; y(x) = 3e^{-x^3}$

Section 2.6 An initial value and its exact solution are given. Apply the Runge-Kutta method to approximate this solution on the interval $[0, 0.5]$ with step size $\Delta x = 0.25$. Construct a table showing five-decimal-place values of the approximate solution and actual solution at the points $x = 0.25$ and 0.5 .

#3 $y' = y + 1, y(0) = 1; y(x) = 2e^x - 1$

#6 $y' = -2xy, y(0) = 2; y(x) = 2e^{-x^2}$