MATH 320, Spring 2013, Assignment 6

Due date: Monday, March 18

Name (printed): ______________________________

UW Student ID Number: ______________________________

Discussion Section: (circle)

Robin Prakash:	**301**	**302**	**303**	
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Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.
2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.
3. Show all the work required to obtain your answers.
4. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

S#	Q#	Mark
3.4	all	/6
3.5	all	/7
—	1	/6
—	2	/6

Total:	/25
Bonus:	/2

Matrix Operations, Inverses

Suggested problems:

Section 3.4: 1-22, 29-32
Section 3.5: 1-22, 29-37

Problems for submission:

Section 3.4: 3, 5, 8, 12, 16, 21
Section 3.5: 6, 13, 22, 30
(Justify your answers for full marks!)

A common application of matrices is to define *linear transformations* on vectors (or points). To transform points in the standard Euclidean plane to other points in the plane we need to define a 2-by-2 matrix $A = [a_{ij}]$ and compute new points according to $\vec{w} = A\vec{v}$.

1. Consider the *rotation matrix* given by

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}.$$

(a) Show that the rotation matrix R preserves the distance of points from $(0, 0)$. That is to say, show that $\sqrt{v_1^2 + v_2^2} = r$ implies $\sqrt{w_1^2 + w_2^2} = r$ for the transformation $\vec{w} = R\vec{v}$.

(b) Show for the square of the rotation matrix R is given by

$$R^2 = \begin{bmatrix} \cos(2\theta) & -\sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{bmatrix}.$$

(c) Show that the inverse of the rotation matrix R is given by

$$R^{-1} = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

(d) Calculate the matrices R, R^2 and R^{-1} for the values (i) $\theta = \pi/4$, (ii) $\theta = \pi/2$, and (iii) $\theta = 7\pi/6$. Use these matrices to compute the transformed points $\vec{w} = R\vec{v}$ for $\vec{v} = [1\ 0]^T$. Does this transformation earn the title of *rotation* matrix? Explain why or why not. [**Hint:** It may help to draw the points in the plane!]

(e) **Bonus!** Show for the rotation matrix R that

$$R^n = \begin{bmatrix} \cos(n\theta) & -\sin(n\theta) \\ \sin(n\theta) & \cos(n\theta) \end{bmatrix}.$$

[**Hint:** Use induction.]

2. A matrix is called a *projection matrix* if $P \neq I$ and $P^2 = P$. (The name derives from the fact that, for $\vec{w} = P\vec{v}$ we have $P\vec{w} = \vec{w}$. This corresponds to the intuition that, if we project a point to a surface or plane, applying the operation multiple times does not move it again.)

   (a) Verify that the following matrices are projection matrices:

   $$\text{(i) } P_1 = \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix}, \text{ (ii) } P_2 = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}, \text{ (iii) } P_3 = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}.$$

   (b) Each of the projection matrices in part (a) project points in the plane onto a line. By computing the projection of a few points (or more sophisticated method), identify the line onto which the projection operations project points for the three matrices in part (a). [**Hint:** It is enough to project two points and determine the line connecting them.]

   (c) (Textbook: Section 3.4, Question #29) Show that every matrix

   $$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

   has the property that

   $$A^2 = (a + d)A - (ad - bc)I$$

   where I is the 2×2 identity matrix.

   (d) Use the result of part (c) to determine conditions which guarantee A is a projection matrix. Verify that the matrices in part (a) satisfy these conditions. [**Hint:** There are two conditions!]