# MATH 320, Spring 2013, Assignment 7
## Due date: Friday, March 22

Name (printed): _____

UW Student ID Number: _____

Discussion Section: (circle)

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|---|---|---|---|
| **Robin Prakash:** | 301 | 302 | 303 |
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| **Raghvendra Chaubey:** | 352 353 354 | 355 | |

## Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.

2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.

3. Show all the work required to obtain your answers.

4. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

| S# | Q# | Mark |
|-----|-----|------|
| 3.6 | 6 | /2 |
| 3.6 | 16 | /2 |
| 3.6 | 21 | /2 |
| 3.6 | 30 | /3 |
| 3.6 | 37 | /3 |
| 3.6 | 52 | /2 |
| ~~4.1~~ | ~~16~~ | ~~/2~~ |
| ~~4.1~~ | ~~17~~ | ~~/2~~ |
| ~~4.1~~ | ~~20~~ | ~~/2~~ |
| — | 1 | /6 |
| Total: | | /20 |

# Determinants, ~~Vector Spaces~~

**Suggested problems:**

Section 3.6: 1-6, 13-53, 59, 60
~~Section 4.1: 1-41~~

**Problems for submission:**

Section 3.6: 6, 16, 21, 30, 37, 52
~~Section 4.1: 16, 17, 20~~
*(Justify your answers for full marks!)*

1. A matrix parameter which often arises in conjunction with the determinant is the *trace*. The trace of an $n \times n$ matrix $A$ is defined as

$$\text{tr}(A) = \sum_{i=1}^{n} a_{ii}.$$

   In other words, it is the sum of the elements along the diagonal. To see one application where the trace arises, consider the following problem (which we will reconsider in more depth in a few weeks).

   Consider an arbitrary $2 \times 2$ matrix $A = [a_{ij}]$. Suppose we are interested in manipulating $A$ by subtracting (or adding) a fixed value from the diagonal (i.e. we are interested in the matrix $B = A - \lambda I$ where $\lambda \in \mathbb{R}$).

   (a) Show that the condition $\det(B) = 0$ gives the *characteristic polynomial*

   $$\lambda^2 - \text{tr}(A)\lambda + \det(A) = 0. \qquad (1)$$

   (The solutions $\lambda$ of the characteristic equation give the possible values of $\lambda$ for which $\det(B) = \det(A - \lambda I) = 0$.)

   (b) Show that a matrix $B$ permits real values $\lambda \in \mathbb{R}$ satisfying $\det(B) = 0$ if and only if $\text{tr}(A)^2 \geq 4\det(A)$.

   (c) Determine the values of $\lambda$ for which $\det(B) = \det(A - \lambda I) = 0$ for the following matrices $A$:

   (i) $A = \begin{bmatrix} -2 & 1 \\ 0 & 2 \end{bmatrix}$, (ii) $A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$, (iii) $A = \begin{bmatrix} 1 & -4 \\ 4 & 1 \end{bmatrix}$.

   [**Hint:** This can be done directly for each matrix, but it may save some time to solve for $\lambda$ in (1) in terms of $\text{tr}(A)$ and $\det(A)$.]