MATH 320, Spring 2013, Assignment 9

Due date: Friday, April 19

Name (printed): ______________________________

UW Student ID Number: ______________________________

Discussion Section: (circle)

Robin Prakash:	**301**	**302**	**303**	
Sowmya Acharya:	**304**	**306**	**307**	**308**
Raghvendra Chaubey:	**352**	**353**	**354**	**355**

Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.
2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.
3. Show all the work required to obtain your answers.
4. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

S#	Q#	Mark
6.1	#4	/2
6.1	#10	/2
6.1	#18	/2
6.1	#34	/3
—	1	/8
—	2	/8
Total:		/25
Bonus:		/2

Eigenvalues and Eigenvectors

Suggested problems:

Section 6.1: 1-37

Problems for submission:

Section 6.1: 4, 10, 18, 34
(Justify your answers for full marks!)

1. Find the eigenvalues and eigenvectors of the following matrices. If necessary, state the eigenvectors in complex form or find the generalized eigenvectors associated with a defective eigenvalue. [For part (c), find all of the eigenvalues but compute the eigenvectors for *real-valued eigenvalues* only. You do not have to compute the eigenvectors for complex-valued eigenvalues, although it is good practice to try!]

$$\text{(a)}\;\; A = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}, \qquad \text{(b)}\;\; B = \begin{bmatrix} 6 & 4 \\ -4 & -2 \end{bmatrix}$$

$$\text{(c)}\;\; C = \begin{bmatrix} 0 & -1 & 1 \\ 5 & 4 & -5 \\ 2 & 1 & -1 \end{bmatrix}.$$

2. The eigenvector equation $A\vec{v} = \lambda\vec{v}$ applies to each eigenvalue/eigenvector pair λ, $\vec{v}$ *individually*, but we can also consider all of the eigenvalues and eigenvectors *simultaneously*. If we have n regular real-valued eigenvectors, we can define the matrices

$$P = \left[\; \vec{v}_1 \mid \vec{v}_2 \mid \cdots \mid \vec{v}_n \;\right], \quad \text{and} \quad D = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$$

and have that $AP = PD$. Furthermore, since the eigenvectors of A are linearly independent, P is invertible and we can write $A = PDP^{-1}$. In other words, we can use the eigenvectors to decompose the matrix A into a form which just depends on the eigenvalues and eigenvectors. This is called the *diagonalization* of A.

(a) Consider the matrices from Section 6.1, Questions #4 and #18. Find the matrices P, D, and P^{-1} and use them to verify that $A = PDP^{-1}$.

(b) Show that any diagonalizable matrix satisfies $A^n = PD^nP^{-1}$ and use this to compute A^2 for the two matrices from part #2(a).

(c) If $A_1, A_2, \ldots, A_k$ are invertible $n \times n$ matrices, it is known that $(A_1A_2 \cdots A_k)^{-1} = A_k^{-1}A_{k-1}^{-1} \cdots A_1^{-1}$. It is also known that $(A^{-1})^{-1} = A$. Use the diagonalization $A = PDP^{-1}$ to construct a formula for A^{-1} and use this formula to find A^{-1} for the matrices from Section 6.1, Question #4 and #18. [**Hint:** You can find D^{-1} by inspection!]

Bonus! If an $n \times n$ matrix has either a complex eigenvalue or a defective eigenvalue, then the formula $A = PDP^{-1}$ does not work. Instead we use the *real Jordan normal form* $A = PJP^{-1}$ where the form of J will depend on the particular case.

(a) For a complex eigenvalue, we know that $\lambda = Re(\lambda) + Im(\lambda)i$ and $\vec{v} = Re(\vec{v}) + Im(\vec{v})i$. If we define

$$P = \left[\begin{array}{c|c} Re(\vec{v}) & Im(\vec{v}) \end{array}\right], \quad \text{and} \quad J = \begin{bmatrix} Re(\lambda) & Im(\lambda) \\ -Im(\lambda) & Re(\lambda) \end{bmatrix}$$

then we have $A = PJP^{-1}$. For the matrix A from Question #1, find P, J, and P^{-1}, and verify that $A = PJP^{-1}$.

(b) For a defective eigenvalue λ, we have a regular eigenvector $\vec{v}_1$ and a chain of generalized eigenvectors $\vec{v}_2, \vec{v}_3, \ldots, \vec{v}_k$. If we define

$$P = \left[\begin{array}{c|c|c|c} \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_k \end{array}\right], \quad \text{and} \quad J = \begin{bmatrix} \lambda & 1 & 0 & \cdots & 0 \\ 0 & \lambda & 1 & \cdots & 0 \\ 0 & 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda \end{bmatrix}$$

then we have $A = PJP^{-1}$. For the matrix B from Question #1, find P, J, and P^{-1}, and verify that $A = PJP^{-1}$.