MATH 320, Spring 2013, Assignment 10

Due date: Friday, April 26

Name (printed): _____

UW Student ID Number: _____

Discussion Section: (circle)

Robin Prakash:	**301**	**302**	**303**	
Sowmya Acharya:	**304**	**306**	**307**	**308**
Raghvendra Chaubey:	**352**	**353**	**354**	**355**

Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.
2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.
3. Show all the work required to obtain your answers.
4. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

S#	Q#	Mark
5.1	all	/7
5.2	all	/4
5.3	all	/8
—	1	/6
Total:		/25
Bonus:		/2

Second-Order Linear Equations

Suggested problems:

Section 5.1: 1-26, 31-42
Section 5.2: 1-20
Section 5.3: 1-38

Problems for submission:

Section 5.1: 15, 20, 25
Section 5.2: 6, 11
Section 5.3: 7, 23, 38
(Justify your answers for full marks!)

1. Consider a spring with a mass of 5 kg. Suppose the restoring force is 125 Newtons per meter of displacement, and the frictional force is 50 Newtons per meter per second of velocity.

   (a) Set up the second-order differential equation governing the motion of the mass on the spring.

   (b) Find the general solution of the differential equation found in part (a). Classify the system as underdamped, critically damped, or overdamped.

   (c) Find the particular solutions for the initial conditions

   (i) $x(0) = 0$ m, $x'(0) = 5$ m/s
   (ii) $x(0) = 5$ m, $x'(0) = 0$ m/s
   (iii) $x(0) = -5$ m, $x'(0) = -25$ m/s

   (d) Sketch the three solutions found in part (d) for $t \geq 0$. Show that the two initial conditions are satisfied for each function. [**Hint:** Computer-generated plots are perfectly acceptable. If you are unsure what the functions look like, they can be googled.]

   (e) Suppose the frictional force was changed to 25 Newtons per meter per second of velocity. Without explicitly solving the differential equation, explain how the qualitative form of the solution will change. Explain why this makes sense in the context of the physical problem.

(f) Suppose the frictional force was changed to 100 Newtons per meter per second of velocity. Without explicitly solving the differential equation, explain how the qualitative form of the solution will change. Explain why this makes sense inthe context of the physical problem.

Bonus! Consider the differential equation

$$y'''(x) - 3y''(x) + 3y'(x) - y(x) = 0.$$

It is easy check that the substitution $y(x) = e^{rx}$ gives $e^{rx}(r-1)^3 = 0$ so that $y_1(x) = e^x$ is a solution. We know, however, that there are *three* linearly independent solutions, not just one.

Use the trick used in class to *construct* the other two linearly independent solutions. [**Hint:** It is sufficient to assume the form $y_2(x) = u(x)y_1(x)$ as we did in class. Both solutions can be obtained from this substitution.]