MATH 320, Spring 2013, Assignment 11

Due date: Monday, May 6

Name (printed): ______________________________

UW Student ID Number: ______________________________

Discussion Section: (circle)

Robin Prakash:	**301**	**302**	**303**	
Sowmya Acharya:	**304**	**306**	**307**	**308**
Raghvendra Chaubey:	**352**	**353**	**354**	**355**

Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.
2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.
3. Show all the work required to obtain your answers.
4. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

S#	Q#	Mark
5.2	24	/2
5.5	10	/3
5.5	16	/3
5.5	26	/2
5.5	34	/3
5.5	38	/4
—	1	/8

Total:	/25

Second-Order Linear Non-Homogeneous Equations

Suggested problems:

Section 5.2: 21-26
Section 5.5: 1-42, 44-46
Section 5.6: 1-18

Problems for submission:

Section 5.2: 24
Section 5.5: 10, 16, 26, 34, 38
(Justify your answers for full marks!)

1. Resonance is not a phenomenon reserved for undamped mechanisms. Re-consider the mass-spring example from class with the additional constraint that the system is subject to 2 Newtons per meter per second of damping. Suppose the system undergoes periodic forcing of the form $\cos(\omega t)$ where ω is as yet undetermined. That is to say, consider the following example:

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 4x(t) = \cos(\omega t). \tag{1}$$

[**Hint:** See Chapter 5.6, Example 6 for help.]

   (a) Find the general solution of (1). [**Hint:** Note that we do not need to consider cases for ω!]

   (b) Suppose the spring is initially at rest at the neutral position $x = 0$. Solve the initial value problem corresponding to (1).

   (c) By considering the limit as $t \to \infty$, divide the solution from part (a) into two parts: a *transient solution* $x_{tr}(t)$ which goes to zero in the limit, and a *steady periodic* solution $x_{sp}(t)$ which does not. (In other words, write $x(t) = x_{tr}(t) + x_{sp}(t)$.)

   (d) Find the amplitude of the steady periodic function $x_{sp}(t)$ found in part (c). [**Hint:** Consider writing the portion $x_{sp}(t)$ in the form $A\cos(\omega t - \alpha)$ but only find A.]

   (e) At which value of ω does A achieve its maximum? Interpret this value in terms of the physical system. In particular, how does it compare to the natural frequency ω_0 of the system? [**Hint:** Take the derivative of A with respect to ω!]