MATH 320, Spring 2013, Assignment 12

Not for submission!

Name (printed): ______________________________

UW Student ID Number: ______________________________

Discussion Section: (circle)

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Instructions

1. Fill out this cover page **completely** and affix it to the front of your submitted assignment.
2. **Staple** your assignment together and answer the questions in the order they appear on the assignment sheet.
3. Show all the work required to obtain your answers.
4. You are encouraged to collaborate on assignment problems but you must write up your assignment independently. **Copying is strictly forbidden!**

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Total:	$/\infty$

First-Order Systems of Differential Equations

Suggested problems:

Section 7.1: 1-10, 21-23
Section 7.3: 1-26
Section 7.5: 1-32

Problems for submission:

Section 7.1: 2, 7
Section 7.3: 5, 12, 24
Section 7.5: 4, 11, 18, 23
(Justify your answers for full marks!)

1. Another way to view the eigenvalue/eigenvector method of solving linear system is to decompose the matrix A into canonical form. If A has a distinct set of real eigenvalues $\lambda_1, \dots, \lambda_n$ and eigenvectors $\vec{v}_1, \dots, \vec{v}_n$, we can decompose A according to $A = PDP^{-1}$ where

$$P = \left[\begin{array}{c|c|c|c} \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_n \end{array} \right] \quad \text{and} \quad D = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}.$$

   (a) Consider the system from Question #5 in Section 7.3. Show that the variable substitution $\vec{y} = P^{-1}\vec{x}$ transforms the system $\dfrac{d\vec{x}}{dt} = A\vec{x}$ into an uncoupled system of the form $\dfrac{d\vec{y}}{dt} = D\vec{y}$.

   (b) Find the general solution $\vec{y}(t)$ of the differential equation $\dfrac{d\vec{y}}{dt} = D\vec{y}$ found in part (a). [**Hint:** Remember the constants of integration!]

   (c) Use the substitution from part (a) to verify the solution of $\dfrac{d\vec{x}}{dt} = A\vec{x}$ already found for Question #5 in Section 7.3.

2. Given a repeated eigenvalue λ and a chain of generalized eigenvectors $\vec{v}_1, \dots, \vec{v}_n$ (where $\vec{v}_1$ is a regular eigenvector), we can decompose the

matrix A into the form $A = PJP^{-1}$ where P is as above and

$$J = \begin{bmatrix} \lambda & 1 & 0 & \cdots & 0 \\ 0 & \lambda & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda \end{bmatrix}.$$

(a) Consider the system from Question #11 in Section 7.5. Show that the variable substitution $\vec{y} = P^{-1}\vec{x}$ transforms the system $\dfrac{d\vec{x}}{dt} = A\vec{x}$ into a system of the form $\dfrac{d\vec{y}}{dt} = J\vec{y}$.

(b) Find the general solution $\vec{y}(t)$ of the differential equation $\dfrac{d\vec{y}}{dt} = J\vec{y}$ found in part (a). [**Hint:** The system is not fully uncoupled, but can be solved by solving for y_3 first, then y_2, then y_1.]

(c) Use the substitution from part (a) to verify the solution of $\dfrac{d\vec{x}}{dt} = A\vec{x}$ already found for Question #11 in Section 7.5.