We do not always have the same number of equations as variables to solve for.

Definition: A linear system of equations is said to be:

~~Over~~ **Under determined:** if # of equations is strictly less than # of variables.

Over determined if # of equations is greater than # of variables

Same method!

Example: Solve

$$\begin{aligned} x_1 - 2x_2 - 5x_3 - 12x_4 &= 1 \\ 2x_1 + 3x_2 + 18x_3 + 11x_4 &= 9 \\ 2x_1 + 5x_2 + 26x_3 + 21x_4 &= 11 \end{aligned}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -2 & -5 & -12 & 1 \\ 2 & 3 & 18 & 11 & 9 \\ 2 & 5 & 26 & 21 & 11 \end{array}\right] \begin{array}{l} R_2' = R_2 - 2R_1 \\ R_3' = R_3 - 2R_1 \end{array} \left[\begin{array}{cccc|c} 1 & -2 & -5 & -12 & 1 \\ 0 & 7 & 28 & 35 & 7 \\ 0 & 9 & 36 & 45 & 9 \end{array}\right]$$

$$\begin{array}{l} R_2' = \frac{1}{7}R_2 \\ \sim \end{array} \left[\begin{array}{cccc|c} 1 & -2 & -5 & -12 & 1 \\ 0 & 1 & 4 & 5 & 1 \\ 0 & 9 & 36 & 45 & 9 \end{array}\right] \begin{array}{l} R_1' = R_1 + 2R_2 \\ \sim \\ R_3' = R_3 - 9R_2 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 3 & -2 & 3 \\ 0 & 1 & 4 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Done!

Set $x_3 = s$, $x_4 = t$

Then

$R_1 \Rightarrow x_1 = 3 - 3s + 2t$

$R_2 \Rightarrow x_2 = 1 - 4s - 5t$