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Math 320, Spring 2013, Term Test II
Linear Algebra and Differential Equations
0.2in Date: Friday, April 12
Lecture Section: 001

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Name (printed): ________________________________ UW Student ID Number: ________________________________

Discussion Section: (circle)

Robin Prakash:	**301**	**302**	**303**	
Sowmya Acharya:	**304**	**306**	**307**	**308**
Raghvendra Chaubey:	**352**	**353**	**354**	**355**

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1. Definitions:

   (a) State
   [1] the definition of what it means for a set of vectors $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ to be linearly independent.

   (b) Give
   [1] the definition of what it means for V to be a *vector space*.

   (c) State
   [1] the Rank-Nullity Theorem.

2. True/False:
   [3]

   (a) If an $n \times n$ matrix A has a left inverse B then the right inverse is the adjoint of B. [True / False]

   (b) An $n \times n$ matrix A is invertible if and only if $\det(A) \neq 0$. [True / False]

   (c) Every basis of a 4-dimensional subspace of $\mathbb{R}^7$ has 4 vectors in it. [True / False]

3. Gaussian Elimination:

(a) Write the following linear system as the matrix equation $A\vec{x} = \vec{b}$ then determine values of $k \in \mathbb{R}$

[4] for which the system has (i) a unique solution; (ii) no solution; and (iii) infinitely many solutions.

$x_1 \; +2x_2 - \; x_3 \; = \; -1 - 2x_1 \; -3x_2 + \; x_3 \; = \; 1 - 3x_1 \; + \; x_2 + kx_3 \; = \; -4.$

(b) Use the fact that

[2]

$$A = \begin{bmatrix} -3 & 1 & 4 \\ -1 & 3 & -2 \\ 3 & -1 & -5 \end{bmatrix} \qquad \Longrightarrow \qquad A^{-1} = \frac{1}{8}\begin{bmatrix} -17 & 1 & -14 \\ -11 & 3 & -10 \\ -8 & 0 & -8 \end{bmatrix}$$

to solve the linear system

$-3x_1 \; + \; x_2 \; + \; 4x_3 \; = \; 1 - x_1 \; +3x_2 \; - \; 2x_3 \; = \; 13x_1 \; - \; x_2 \; - \; 5x_3 \; = \; 1.$

4. Determinants and Inverses: Determine

[4] the adjoint of the following matrix and use it to find A^{-1} (part marks will be awarded for solving for A^{-1} by another method):

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 2 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix}$$

5. Matrix Spaces: Consider

[3] the following matrix A and its row-reduced echelon matrix (to the right):

$$A = \begin{bmatrix} 1 & 1 & 3 & -2 \\ 2 & 0 & -4 & 1 \\ 1 & -1 & -7 & 0 \\ -1 & 2 & 12 & 7 \end{bmatrix} \qquad \longrightarrow \qquad \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

State a basis for the following vector spaces:

(a) The row space of A:

(b) The column space of A:

(c) The null space of A:

6. Eigenvalues/Eigenvectors: Determine

   [4]the eigenvalues and eigenvectors of the following matrix:

$$A = \begin{bmatrix} 1 & -4 \\ -2 & -1 \end{bmatrix}$$

7. Theory: Suppose

   [2]$V = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\} \subseteq \mathbb{R}^n$ are pairwise linearly independent (i.e. we have $\{\vec{v}_1, \vec{v}_2\}$, $\{\vec{v}_1, \vec{v}_3\}$, and $\{\vec{v}_2, \vec{v}_3\}$ are linearly independent). Prove or disprove the claim that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent. [**Hint:** To prove, you must show the claim holds; to disprove, find a counter example.]

THIS PAGE IS FOR ROUGH WORK