Math 521, Spring 2014, Term Test I

Analysis I

Date: Friday, February 21

Time: 9:55-10:45 a.m.

Lecture Section: 001

Name (printed): ______________________________

UW Student ID Number: ______________________________

Instructions

1. Fill out this cover page.
2. Answer questions in the space provided, using back page for overflow and rough work.
3. Show all work required to obtain your answers.
4. Unless otherwise stated, you may use any theorem or result derived in class.

FOR EXAMINERS' USE ONLY	
Page	Mark
2	/6
3	/5
4	/5
5	/4
Total	/20

[3] 1. Definitions:

(a) Let P and Q be statements which have a truth value and suppose that $P \implies Q$. State the *contrapositive*.

(b) Suppose X is an ordered set and $S \subseteq X$ is nonempty and bounded below. Define what it means for $\alpha = \inf(A)$.

(c) State the multiplication axioms for a set $\mathbb{F}$ to be a field.

[3] 2. True/False:

(a) The converse of a statement $P \implies Q$ is never true. [True / False]

(b) For any element $x \in \mathbb{F}$, $x \neq 0$, in any field $\mathbb{F}$, the multiplicative inverse x^{-1} is unique. [True / False]

(c) Suppose S is a finite set. Then $\mathcal{P}(\mathcal{P}(\mathcal{P}(S)))$ is a finite set. [True / False]

[3] 3. Set Proofs (Note: Venn diagrams are a helpful aid but do not constitute a proof!)

Let A and B be sets. Prove that $A \cup B \subseteq A \cap B \implies A = B$.

[2] 4. Ordered Sets

Suppose X is an ordered set and $S \subseteq X$ is bounded above. Prove that if $\sup(S)$ exists, it is *unique*. (i.e. Prove that $\alpha = \sup(S)$ and $\beta = \sup(S)$ implies $\alpha = \beta$.)

[5] 5. Fields

Let $\mathbb{F}$ denote a field. Suppose $x, y, z \in \mathbb{F}$. Using the field axioms and any field result from class other than the one stated, prove the following (you may use each result you prove thereafter, if applicable):

(a) $x \cdot y = 0$ implies that $x = 0$ or $y = 0$

(b) (**Difference of squares**)
$x^2 - y^2 = (x + y) \cdot (x - y)$ (where $x^2 = x \cdot x$ and $x - y = x + (-y)$)

(c) $x^2 = y^2$ implies $x = y$ or $x = -y$ (**Hint:** Consider parts (a) and (b)!)

[4] 6. Countability

Let $\bar{\mathbb{C}}$ denote the set of all values $x = a + bi$ where $a, b \in \mathbb{Q}$, $\sqrt{a^2 + b^2} \leq 1$, and $i = \sqrt{-1}$. Is $\bar{\mathbb{C}}$ countable or uncountable? Prove your claim.

THIS PAGE IS FOR ROUGH WORK