Math 521, Spring 2014, Term Test I

Analysis I

Date: Friday, February 21

Time: 12:05-12:55 p.m.

Lecture Section: 002

Name (printed): ______________________________

UW Student ID Number: ______________________________

Instructions

1. Fill out this cover page.
2. Answer questions in the space provided, using back page for overflow and rough work.
3. Show all work required to obtain your answers.
4. Unless otherwise stated, you may use any theorem or result derived in class.

FOR EXAMINERS' USE ONLY	
Page	Mark
2	/6
3	/5
4	/5
5	/4
Total	/20

[3] 1. Definitions:

(a) Let P and Q be statements which have a truth value and suppose that $P \implies Q$. State the *contrapositive*.

(b) State what it means for an ordered set X to have the least-upper-bound property.

(c) State the multiplication axioms for a set $\mathbb{F}$ to be a field.

[3] 2. True/False:

(a) The converse of a statement $P \implies Q$ is never true. [True / False]

(b) In any field $\mathbb{F}$, the additive identity $0 \in \mathbb{F}$ is unique. [True / False]

(c) Suppose S is a finite set. Then $\mathcal{P}(\mathcal{P}(\mathcal{P}(S)))$ is a finite set. [True / False]

[3] 3. Set Proofs (Note: Venn diagrams are a helpful aid but do not constitute a proof!)

Let A and B be sets. Prove that $A \cup B \subseteq A \cap B \Longrightarrow A = B$.

[2] 4. Countability

Suppose $\{S_n\}$ is a countably infinite family of countably infinite sets $S_n, n \in \mathbb{N}$. Prove that

$$S = \bigcup_{n=1}^{\infty} S_n$$

is countably infinite.

[5] 5. Fields

Let $\mathbb{F}$ denote a field. Suppose $x, y, z \in \mathbb{F}$. Using the field axioms and any field result from class other than the one stated, prove the following (you may use each result you prove thereafter, if applicable):

(a) $x \cdot y = 0$ implies that $x = 0$ or $y = 0$

(b) (**Difference of squares**)
$x^2 - y^2 = (x + y) \cdot (x - y)$ (where $x^2 = x \cdot x$ and $x - y = x + (-y)$)

(c) $x^2 = y^2$ implies $x = y$ or $x = -y$ (**Hint:** Consider parts (a) and (b)!)

[4] 6. Ordered Sets

Suppose that X is an ordered set and every nonempty $S \subseteq X$ which is bounded below has the property that $\inf(S) \in X$. Prove that X has the least-upper-bound property. (**Note:** You must prove this directly. You *may not* use the result derived from the least-upper-bound property obtained in class!)

THIS PAGE IS FOR ROUGH WORK