Math 521, Spring 2014, Term Test II

Analysis I

Date: Friday, April 4

Time: 12:05 - 12:55 p.m.

Lecture Section: 002

Name (printed): ______________________________

UW Student ID Number: ______________________________

Instructions

1. Fill out this cover page.
2. Answer questions in the space provided, using back page for overflow and rough work.
3. Show all work required to obtain your answers.
4. Unless otherwise stated, you may use any theorem or result derived in class.

FOR EXAMINERS' USE ONLY	
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Total	/20

[3] 1. <u>Definitions:</u>

(a) Suppose (X, d) is a metric space and $S \subseteq X$. Define what it means for $x \in X$ to be

(i) an interior point of S.

(ii) a boundary point of S.

(b) State the Bolzano-Weierstrass Theorem.

[2] 2. <u>True/False:</u>

(a) The unit ball $B_1(\mathbf{0})$ in every metric on $\mathbb{R}^n$ contains the same points. [True / False]

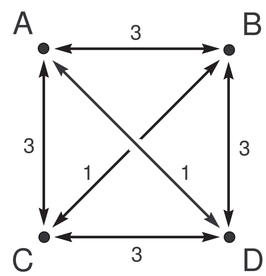
(b) An infinite intersection of closed sets is always closed. [True / False]

(c) Every connected set is closed. [True / False]

(d) If $\lim_{n\to\infty} p_n = p$ for a sequence $\{p_n\} \subseteq S$, then p must be a limit point of S in the topological sense. [True / False]

[2] 3. Metric Spaces

Consider the set $X = \{A, B, C, D\}$ with distances $d : X \times X \mapsto \mathbb{R}$ defined symmetrically by the figure below (and $d(A, A) = d(B, B) = d(C, C) = d(D, D) = 0$):



Is (X, d) a metric space? Justify your answer.

[3] 4. Topology

Let (X, d) be a metric space. Prove that if $A \subseteq X$ and $B \subseteq X$ are open, then $A \cap B$ is open.

5. Compact Sets

[3] (a) Consider the metric space $(\mathbb{Q}, d)$ with $d(x, y) = |x - y|$. Consider the set

$$S = \{x \in \mathbb{Q} \mid -1 \leq x \leq 1\}.$$

Construct an open cover of S which does not have a finite subcover. Justify your choice.

[3] (b) Let (X, d) be a metric space and $S \subseteq X$ be compact. Prove that S is bounded.

[4] 6. Convergence

Use the definition of convergence to prove that

$$\lim_{n\to\infty} \sqrt{n^2 - 1} - n = 0.$$

THIS PAGE IS FOR ROUGH WORK